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A MICROECONOMETRIC STUDY OF AGRICULTURAL DEVELOPMENT

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It is the fundamental premise of this study that agricultural development can be fully understood and effectively planned only by accounting for microeconomic aspects of technology, decision strategies and market environment that play a crucial role in determining economic performance. We call these the strategic details of development.

Economic theorists have often concentrated on global issues of economic development: the long run path of per capita incomes, the existence and character of balanced growth, the intertemporal optimality of alternative growth trajectories. This has led to macroeconomic theories characterized by a few relatively simple, but dramatic properties such as the "iron law of wages" that derived from classical reasoning, or the currently fashionable "golden rules of economic growth." But policymakers and the rank and file civil servants who are charged with implementation, have long known that an awareness of the "big issues" is not sufficient by itself to guide the host of individual decisions for which they are directly responsible or over which they hope to hold sway through well conceived direct and indirect controls. They have found that sooner or later policies must account for the realities of decision-making in the field and factory. Unfortunately for them, however, at this microeconomic level, little guidance can be

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obtained from the traditional economic literature. There has been a wide gap between the principles of macroeconomic development theory and the practice of policy makers and administrators.[1]

During the last decade a new breed of pragmatic model builder has entered the field armed with input-output, mathematical programming and computer simulation techniques of analysis.[2] Most of this new work has emphasized intersectoral structure and has been motivated by policy issues dealing with resource allocation and factor mobility among various industrial sectors and agriculture. The concentration of this type of interdependence has meant that the details of technology and decision-making within individual sectors have had to be subsumed so that the policy maker is still left without much guidance for the planning level where the policies he develops must ultimately be put to work (and, moreover, where his success or failure will ultimately be judged by the people). This study is an attempt to help fill this remaining gap, at least in part, by developing and testing a microeconometric model that is capable of simulating the performance of an individual sector -- in this case, agriculture, in a way that explicitly accounts for the essential features of technology and decision-making.

Part 1 of our paper discusses the general requirements for a dynamic, microeconomic model of agricultural development. Part 2 then presents a mathematical theory that incorporates what we think are the essential features or strategic details of the process. In part 3 this theory is approximated by an operational model that can be estimated and simulated within existing data and computational limitations. Part 4 is devoted to testing the model's ability to describe recent agricultural history in the Central Punjab of India. The paper concludes with a brief list of applications.

Our first purpose has been to improve our understanding of the development process by relating past trends to an explicit representation of agricultural technology and farm decision-making in developing agriculture. Our second purpose is to aid the formulation of effective development policy by projecting likely future time paths of key variables given specific assumptions about various exogenous variables, including those that can be effectively controlled by governmental decision-makers. The present operational model is tailored to the Indian Punjab. With suitable modifications it should be applicable to virtually any region undergoing a transition from traditional to modern agriculture.

1. THE STRATEGIC DETAILS OF DEVELOPMENT[3]

Until recently it was argued by many, and with great force, that people in various societies behave according to rules so different that micro-economic theory is not relevant, that the people of less developed countries are tradition bound, that cultural and institutional restraints severely circumscribe their responsiveness to market incentives, and that the developed countries have a kind of monopoly on "economic man."[4] SCHULTZ [1964] on the other hand argued that traditional patterns were maintained not because of hidebound restraints but because they represented a rational equilibrium under existing conditions. His position has been confirmed by the growing number of supply response studies in the LDC's.[5]

Focusing on the question of whether or not peasants in traditional or near traditional agriculture respond to opportunities which are made available by changes in market conditions, they have shown that agricultural production of specific commodities in specific LDC's is price responsive, especially when adjustment lags due to uncertainty and quasifixity of capital stocks are accounted for. Moreover, they suggest that the general form and direction of this response is consistent with price theory and that peasants in traditional agriculture respond to market incentives when sufficient incentives exist. It is indeed on the basis of these results that we believe behavior of farmers in the LDC's can be represented by a model of economic man, by whom we mean a man whose choices among well defined alternatives are made in an attempt to maximize the attainment of well defined goals. Our model incorporates this explicitly. It seems, however, that the conventional marginal analysis does not adequately describe the application of this maximizing principle in reality by peasant farmers, who continue to play an important role in the decentralized decision-making structure of most countries in the Third World. At least six complications -- strategic details of farm decision-making -- must be incorporated into the analysis. These are the interdependence of farm household and firm decisions, multi-product, multi-process technology, uncertainty, technological change, learning and nonfarm linkages.[6] We shall comment briefly on these in turn.

(1) Interdependence of farm household-firm decisions.

Economists have traditionally simplified the overall economic allocation problem into two separate parts: the household income allocation problem, described by constrained utility maximization, and a firm resource allocation problem described by profit maximization. Nowhere in this theoretical tactic more clearly expounded than in KOOPMANS [1957] where the principles are illustrated with the "time-honored example of a man by whom production and consumption decisions are made in combination: Robinson Crusoe..." who in the course of the analysis is shown to be decomposable into Robinson the producer and Robinson the consumer.

Defoe's Crusoe is not merely a convenient literary illusion. He is the prototype of the "peasant" or "family" farmer found in virtually every agricultural region in the world. But while, for the sake of simplicity, the farm decision is no doubt broken up into smaller, more manageable parts in practice and while we shall indeed exploit a given decision decomposition hypothesis below, it does violence to reality to suppose that the decomposition takes place on the farm as it does in the nonfarm economy. Some authors have recognized the fundamental interdependence in the farm between firm and household decisions. HEADY, BACK and PETERSON [1953] were among the early investigators to quantify this interdependence. More recently NAKAJIMA [1957, 1963 and 1965] and MELLOR [1964, 1966] have contributed to a clearer theoretical understanding of this interdependence in the context of the less developed countries. It is now time to incorporate this feature in an empirical model of production response in traditional agriculture. KRISHNA [1965] has made a step in this direction by deriving a marketable surplus supply function from a mathematical version of Nakajima's analysis. Our model represents another, somewhat more elaborate step.

(2) Farm Technology.

The neoclassical analysis of the firm is for the most part based on twice differentiable production functions which are usually assumed to involve a single output and which represent a given production technology. Contrastingly, agriculture is really characterized by multiple outputs, and during periods of transition (which constantly occur), by multiple technologies. Activity analysis, as developed by KOOPMANS [1951], LEONTIEF et al [1953] and applied by many investigators can accomodate all three of these characteristics in any amount of detail.

Direct observation leads us to appreciate the fact that traditional agriculture is a complex phenomenon with hundreds of individual tasks being performed, in many possible combinations, requiring detailed knowledge of soils, climate, topography, and with scarce resources being distributed over time and crop use. Choices among these many tasks are merely enlarged when new implements, power sources, and materials are introduced. We do not argue that it is necessary for the purposes of development policy to accomodate all of the details with which the peasant himself must contend. We do believe that many of them are important and that only by representing major technological alternatives in an activity analysis framework can agriculture be effectively understood and planned -- at any level.

(3) Uncertainty.

The fact that farming is highly uncertain in many of its aspects is obvious to a casual observer. Accounting for it in some way is a virtual necessity for the farmer and if he is to understand agriculture a necessity for the economist as well. It seems doubtful, however, that the farmer's decision strategies are the same as those used by sophisticated gamblers in St. Petersburg or Monte Carlo. It seems likely instead that his strategies come closer to rules that might be summarized as _strategies of cautious optimizing_. Examples are the behavioral bounds of CYERT and MARCH [1963], the focus-loss principle of SHACKLE [1958], the chance constrained programming models of CHARNES and COOPER [1959], and the safety first principle of ROY [1952]. We have taken this latter point of view and as a first approximation, have adopted a particular representation of it elaborated by one of us elsewhere, DAY [1970], [1971].

(4) Technological Change.

The principles just outlined when properly constructed would be quite consistent with, indeed would help explain a state of economic equilibria in traditional agriculture, a state according to SCHULTZ [op. cit.] in which, given the state of the arts, the rates of return to traditional inputs are so low that little or no net investment takes place, and in which comparatively few significant inefficiencies in the allocation of the factors of production exist. In such a state he argues small changes in either the relative prices of inputs or in the quantities of inputs unchanged in quality are unlikely to bring about any long run departure from this equilibrium. As a result, only new technology can shift agriculture from this traditional state.

Within the activity analysis framework at least four specific components of "new" or nontraditional technology should be considered: new materials, new implements and power sources, and new cultural practices. Activities involving these and traditional activities, accomodated within the set of possible farm operations enable the many choices describing the transition from traditional to modern agriculture to be analyzed.

(5) Learning and Adoption.

The breakdown of age old practices takes time partly because the supply of new inputs must go through a development process of its own. This places external constraints on adoption of new technology, a factor no doubt of great importance. In addition, adoption is internally constrained by a learning process which proceeds as more and more farmers gain familiarity with and confidence in their ability to successfully exploit the new opportunities. The impact of new technology, following upon its innovation is thus distributed over time, a fact that should clearly be a part of a complete analysis of development, and a further complication to be incorporated in a model of an agricultural region based on the principle of economizing.

(6) Nonfarm linkages.

We have mentioned the external constraints imposed by limited supplies of nonfarm inputs such as industrially produced implements, machines, materials, and fuel. This means that development takes place within a multi-sectoral context. Several additional nonfarm linkages are crucial. These involve the supply of credit, the supply of wage-labor, and the demand for final products. Some of these linkages occur indirectly through market prices, and some occur directly through physical and behavioral limitations on the use and availability of resources. Hence, even in models that focus almost entirely on development and planning within the sector these strategic linkages must be accounted for.

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The reader uninterested in the technical development of the ideas just adumbrated, and sufficiently confident in our ability to incorporate them into a quantitative framework can proceed from this point to section 4.2 below where evidence is presented in favor of their explanatory power.

2. THEORY

Having before us six categories of strategic details it should be clear that a complete understanding of agricultural development must involve first, an analysis of how development takes place within the farm sector, and second, a multisectoral analysis of economic development as a whole. It is beyond the scope of the present undertaking to meet both of these requirements. We concentrate here on the first of these, using abstract notation and some new theoretical results that make possible its exegesis with economy of symbols. We then go into the detailed operational representation of the theory in Part 3.

2.1 Decisions

2.1.1 Farm Activities

Farm activities include production, sales, investment, financial and household activities. Let $\mathcal{X}$ be the complete set of these activities. We shall denote an activity by its name or index and equate $\mathcal{X}$ with the set of such names or indexes. Hence $j \in \mathcal{X}$ denotes the name or index of activity j. The intensity with which a given activity is operated we call an activity level and denote it x_j, $j \in \mathcal{X}$. The units depend on the activity in question. Most production activity levels are measured in hectare units, other are in units of volume or weight, some in monetary units. All of them indicate the planned intensity to be operated within a given year with the plans drawn up at the beginning of the year. The vector $x = (x_j)_{j \in \mathcal{X}}$ denotes an n vector of activity levels and we shall call it a decision vector. It belongs to the n dimension decision space $X = \mathbb{R}^n$.

The choice of farm activities for a given year is constrained by three categories of relations: technological, financial and learning.

The first category involves labor, land, commerical input and machine capacity constraints. These define a set T of <u>technologically feasible</u> decision vectors. The second category involves working capital and availability, borrowing limitations, and debt repayment requirements. These define a set F of <u>financially feasible</u> decisions vectors. The third category represents the constraining effects of learning on the adoption of new techniques and leads to a set L of decision vectors <u>compatible with learning</u>. We shall describe the specific structure of these sets in section 3. At this point we need to recognize their dependence on data germane to the decisions for a given year. Each set depends on two types of coefficients which we call <u>constraint</u> and <u>limitation coefficients</u>. Denoting these by vectors B and c respectively and using a superscript t for a discrete time index, we may write

(1) $$T^t = T(B_T^t,\ c_T^t)$$

(2) $$F^t = F(B_F,\ c_F^t)$$

(3) $$L^t = L(B_L,\ c_L^t)$$

Letting $B^t = (B_T^t,\ B_F^t,\ B_L^t)$ and $c^t = (c_T^t,\ c_F^t,\ c_L^t)$ the region of feasible decisions for a given production period may be denoted

(4) $$\Gamma(B^t, c^t) = T(B_T^t,\ c_T^t) \cap F(B_F^t,\ c_F^t) \cap L(B_L^t,\ c_L^t) \ .$$

It is the set of decision vectors that satisfy simultaneously the technical, financial and learning constraints.

2.1.2 Farm Goals and Lexicographic Utility

We assume that the farm has four goals arranged in an absolute priority order. We assume also that these goals can be represented by four real-valued objective functions $\mu_i : X \to \mathbb{R}$ where $\mu_i(x,a^i)$ gives the level of satisfaction of the i^{th} goal given by the decision vector X and where a^i is a vector of parameters. These goals are

μ_1 = the goal of satisfying subsistence consumption;

μ_2 = a goal representing a preference ordering amongst alternative current cash consumption and future income streams;

μ_3 = a metric defining the distance of a given choice from a set of safe-enough choices Z, i.e. $\Phi(x) = \text{dist}(x,Z)$;

μ_4 = net cash returns or profit function.

We let σ_i be a "satisficing level" for the i^{th} goal and let $\varphi_i : X \to \mathbb{R}$ be defined by

$$(5) \qquad \varphi_i(\cdot, a^i, \sigma_i) = \min \{\mu_i(x,a^i), \sigma_i\}, \quad i = 1,\ldots,4.$$

We now suppose that the farmers' plans can be represented by the maximizing $\varphi_1,\ldots,\varphi_4$ in priority order, subject to technical, financial and learning constraints.[7]

The first goal seems reasonable, and relevant in regions where a major part of production is produced for home consumption. The second goal is a device for simplifying the total decision problem. It is structured so as to represent the allocation of cash resources between consumption and saving. The optimum allocation of cash saved is then determined by maximizing goal 4, while the optimum allocation of consumption expenditures amongst individual items is assumed to be determined by maximizing a fifth objective function unspecified in this study.

Goal 3, the safety metric, represents behavior according to a principle of cautious optimizing very much like the safety-first principle or chance-constrained programming. It is more general than those, in that it does not require the specification of any subjective probabilities. It covers unpleasant contingencies other than those covered by the subsistence goal and is meant to represent a strategy to protect the farmer against uncertain but highly damaging feedback effects of extreme departures from previously experienced and successful behavioral patterns. Of course if behavior to guarantee subsistence requires it, extreme departures from past experience are predicted according to the maximization of the subsistence goal. However, given satisfaction of the first two goals, caution plays a role in limiting response to shortrun profit opportunities as incorporated into goal 4.[8]

2.1.3 Farm Decisions as an L* Program.[9]

These hypotheses amount to maximizing a Lexicographic* or L* utility function subject to constraints. Let $\Psi_o = \Gamma(B^t, c^t)$. Then we have the L* programming problem

$$\pi_i^t = \max_{x^t} \{\varphi_i(x^t, a^{it}, \sigma_{it}) \mid x^t \in \Psi_{i-1}\}, \quad i = 1, \ldots, 4. \tag{7}$$

where

$$\Psi_i = \{\varphi_i(x^t, a^{it}, \sigma_{it}) \geqq \pi_i^t\} \cap \Psi_{i-1}, \quad i = 1, \ldots, 4. \tag{8}$$

is the set of choices maximizing the i^{th} goal given that they are feasible and that they maximize (or satisfy) the higher order goals.

2.1.4 Super Utility

This scheme (7)-(8) is called a <u>fourth order weak L* program</u>.[10] It has been shown elsewhere, that this decision-making procedure is equivalent to ordinary constrained utility maximization in the following sense: given certain conditions there exists a super-utility function say $\varphi : X \to \mathbb{R}$ with parameters $a^t = (a^1, \sigma_1, a^2, \sigma_2, a^3, \sigma_3, a^{4t})$ such that the set of solutions

$$(9) \qquad \Psi(a^t, B^t, c^t) = \{x^t \mid \varphi(x^t, a^t) \geqq \pi^t\} \cap \Gamma(B_t, c_t)$$

to the ordinary program

$$(10) \qquad \pi^t = \max_{x^t} \{\varphi(x^t, a^t) \mid x^t \in \Gamma(B^t, c^t)\}$$

is exactly $\Psi_4(a^t, B^t, c^t)$ of (8).

This super-utility function represents a preference ordering over activities which accounts for all of the farmer's considerations of subsistence, commercial consumption, safety and profit goals, in their priority order. Though it may exist, however, this function is probably too complicated to use operationally. The L* approach used here is an alternative, convenient way for proceeding to accomodate these several considerations in a quantitative analysis in a way that is consistent with the conventional concept of economic rationality. It serves as a theoretical basis for specifying the constraint structure of the operational model.

In theory the set $\Psi(a^t, B^t, c^t)$ is in general non-unique. While selection amongst these possible best choices could be explained by a variety of plausible hypotheses we use here algorithmic selection, i.e. choice determined by the first point in Ψ obtained by our computer code. Since in fact Ψ is often single-valued this is not necessarily a restrictive assumption. However, to complete the model we must define a selection operator, we denote it R so that the theoretical prediction of farm plans in year t is

$$(11) \qquad x_t^* = R \cdot \Psi(a^t, B^t, c^t)$$

Since the realized value of the data a^t, B^t, c^t may differ from those upon which the initial plan is based, plans may be modified in reality. We have not tried to account for such short-run planning revisions in our model, but have instead used x_t^* as our estimate of actual behavior.

2.2 Feedback and the Complete Model

2.2.1 Feedback

The data vectors (a^t, B^t, c^t) on which depend decision vectors for a given year depend themselves on previous decisions, previous data and on exogenous variables linking the farm situation to its "external" environment. Satiation levels σ_1^t and σ_2^t may depend on past subsistence and commercial consumption activity levels while[11] the desired safety level σ_3^t may depend on new information on price and income variability.[12] Resource limitations c_T^t depend on past investment activities, while financial bounds depend on previous expenditures, and borrowing activities. Learning proceeds with experience so that learning limitation coefficients c_L^t will depend on previous utilization of "new" technologies. Price expectations based on lagged pricing enter the profit objective and in various coefficients of the financial constraints. Other variables representing the state of the outside economy may be included in calculations of relevant planning data. These observations lead us to recognize the feedback effect of past behavior on current plans and the linkage of farm sector to the nonfarm economy.

2.2.2 Data Present and Past.

All of the data of the decision operator Ψ can be conveniently summarized by the single vector $w^t = (a^t, B^t, c^t)$ so that (11) becomes

$$(12) \qquad x^{*t} = R \cdot \Psi(w^t) .$$

To define the dependence of this datum on past decisions and past data we adopt the following convention. Let v^t be an arbitrary n-vector. Then

$$\mathop{v}_{i=1}^{s}{}^{t-i} = (v^{t-1}, v^{t-2}, \ldots, v^{t-s})$$

is a sn-vector with s component n-vectors $v^{t-1}, \ldots, v^{t-s}$.

The feedback effects summarized in § 2.2.1 can now be represented by the notation

$$(13) \qquad w^t = \omega \, [\mathop{x}_{i=1}^{\tau}{}^{*t-i}, \; z^t]$$

where z^t is a vector of exogenous variables not explained by the theory but representing linkages with the nonfarm-sector (and possibly including lagged exogenous variables).

"ω" is a vector of functions each element of which defines the dependence of one data parameter on past decisions and exogenous variables. Of course many if not most of these will be constant functions meaning that the coefficients are constant. But the notation is general enough to accomodate many types of realistic feedback effects and outside influences.

2.2.3. The Complete Model

Our theory can be briefly summarized by equations (12) and (13) which combined yield the following discrete time dynamic system

$$(14) \qquad x^{*t} = R \cdot \Psi \, [\omega(\mathop{x}_{i=1}^{\tau}{}^{*t-i}, z^t)] .$$

This system is a set of simultaneous τ^{th} order difference equations of a complicated and highly nonlinear nature. It represents current decisions by a decision operator depending on considerations of technology, finance, learning, subsistence, commercial consumption, safety and profits. This decision operator involves choosing amongst feasible alternatives according to a hierarchy of goals on the basis of data that depends on previous decisions and outside influences. It is a microeconomic representation of farm behavior that incorporates in a theoretically consistent manner the strategic details of farm development for which it was our purpose to account.

2.3 Aggregation

We have gone to the trouble of constructing a theory of farm behavior not because we are interested primarily in the fortunes of individual farmers but because we have felt that on a detailed understanding of their behavior would depend an adequate explanation of economic development in the sector as a whole. Obviously, however, it is impossible to derive regional aggregates by adding up predictions for each farmer. Instead, we use the structure specified by (14) to define a regional model to be used for explaining and projecting various regional variables.

The theory of aggregation required to go from the micro level to a regional aggregate is complex and only partially developed. It can not be gone into here. We proceed, however, on the following assumptions. Let w^{ti}, x^{*ti}, Ψ^i, ω^i be the data and decision vectors and the decision and feedback operators respectively for the i^{th} farmer. Let W^t be a regional aggregate of the data vector w^{it} involving suitable weighted averages and aggregates. Assume X^t is the total of the activity levels for the region as a whole. Let Ψ and ω be regional behavior and feedback operators respectively whose structure is identical to the Ψ^i and ω^i, i.e. we assume that $\Psi^i = \Psi$, $\omega^i = \omega$ for all farms.

The regional farm sector model is then

$$(15) \qquad X^t = R \cdot \Psi(W^t)$$

$$(16) \qquad W^t = \omega[\underset{i=1}{\overset{\tau}{X^{t-1}}}, Z^t]$$

where it is assume that

$$(17) \qquad \Psi(W^t) = \sum_{i=1}^{q} \Psi^i[w^{it}] \; .$$

Such a region is <u>aggregatable</u> and allows individual decision units to be subsumed. The assumptions required for (17) to hold are very strong and would not be true necessarily even if the theory of equations (1) - (10) were exactly true -- which it is not -- for each farm. Consequently, a model based on (15) - (16) can at best only be an approximate theory of behavior at the sector level.

2.4 Implications

Before turning to empirical matters let us pause to consider how this theory represents the development process. Given initial conditions of low or nonexistent capacity in highly productive technology, farm behavior will be dominated by subsistence goals. If external demand conditions and internal productivity permit it, commercial sales will lead to cash income for which consumption and farm investment will compete. Subsistence considerations will be gradually pushed into the background. As cash farming grows in importance caution in response to market forces and profit maximizing will come to dominate farm production and investment decisions. Depending on the initial situation farmers might adopt new technology rapidly or in some cases not at all. Indeed, many alternative histories are possible in such a model, with many different phases or stages of development arranged in many possible alternative sequences.

Equilibrium at a stationary state might come about in the absence of technological change, though nothing in the theory guarantees that possibility. Indeed such empirical evidence as we now have suggests that agriculture in very diverse situations is inherently unstable once commercial farming activities become important.[13] The cause seems to lie in the highly inelastic demand for agricultural produce and its feedback effect through price and working capital supplies.

The investigation of the existence of stationary states, their stability or instability, and the possibility and character of multiple phase and even indeterministic solutions to theoretical systems of the type (15) - (16) has been begun and the interested reader is referred elsewhere for a further discussion of these matters.[14]

However one property of this theory of such great importance that we should comment on it before proceeding is its incomplete determinancy in the following sense. We specified a selection operator which we acknowledged to be more or less arbitrary: after the goals that rationally might be pursued in the L* program there remains an indeterminant residual of choices, the contents of the set Ψ. Even if this set contains more than one member only infrequently (as we suggested would be the case) the element of incomplete causality clearly remains. The implication is that from time to time decision-makers' choices may be arbitrary -- perhaps random or unpredictable -- and hence the evolution of society imperfectly predictable as well. At best society's behavior would seem to be predictable within bounds.

On the basis of this consideration we should be highly surprised if our operational model predicts actual history with extreme accuracy. This causal imcompleteness is fundamental to the theory[15] and not the result of aggregation errors due to the failure of the assumptions behind equations (15) - (16). Adding the latter source of error to the former we are led to take the position that approximate accuracy of our model in explaining the past is a <u>very strong</u> confirmation of its fundamental validity, just as it is insufficient grounds to believe that projections based on it will have more than a crude (though perhaps highly valuable) contribution to policy.

3. AN OPERATIONAL FARM SECTOR MODEL

We now specify an operational analogue of the theory just developed. By "operational" we mean not merely capable of being tested experimentally under "ideal circumstances" SAMUELSON [1948, p. 4], but rather -- capable of being tested with data available now and capable of being simulated on contemporary computers. It is not possible to more than outline the model here. A detailed exposition is in SINGH [1971].

3.1. Feasible Decisions

We assume that all activities are measured for the regional aggregate, that they are linear and finite in number

and that each is identified by a unique arbitrary index j belonging to a set of indexes X. Likewise we assume a finite set of constraining "factors" each member of which is identified by a unique arbitrary index i belonging to a set Y. The technical b_{ij} coefficients are assumed to be constant over time (all technology is assumed to be embodied) and are defined so that

$b_{ij} > 0 \Rightarrow$ factor i is a net "input" to activity j,

$b_{ij} = 0 \Rightarrow$ factor i is not involved in activity j,

$b_{ij} < 0 \Rightarrow$ factor i is a "net output" of activity j.

The limitation C_i coefficients are defined at the regional level and follow the rules

$C_i > 0 \Rightarrow$ factor i forms an upper bound on activity combinations,

$C_i = 0 \Rightarrow$ factor i forms a "balance" constraint,

$C_i < 0 \Rightarrow$ factor i forms a lower bound on activity combinations.

It is convenient to define various subsets of activities and constraining factors as follows.

Production activities, $j \in P$, include land preparation, planting, cultivating, fertilizing, harvesting, processing, and transporting. These are distinguished where relevant by type of soil, by type of technology (irrigated, unirrigated, fertilized, unfertilized, bullock, tractor, etc.), by crop and by season (summer and winter). Household activites, $j \in H$, include subsistence, food consumption, commercial consumption, and labor "supplying" on and off farms. Purchase activities, $j \in B$, include the purchase of variable inputs such as fuel, fertilizer, improved seeds, etc. Investment activities, $j \in I$, include land development and the purchase of capital goods such as tractors, motors, implements, bullocks, camels, etc. Financial activities, $j \in F$, include saving, borrowing and debt repayment. Sales activities, $j \in S$, are included for each commercial crop.

Labor constraints, $i \in W$, include exogenous supplies of village wage labor, regional labor and national labor. These supplies are to be augmented by household activities which supply family labor in various amounts by season. Farm family labor supplies are limited exogenously in the current model by the number of farm families, though we hope in the future account for these variables endogenously. Material constraints, $i \in E$, allow for the exogenous specification of regional supplies of electricity, fertilizer, herbicides and insecticides limiting material purchase activities. Land supplies, $i \in L$, in some categories can be augmented by investment in land development (irrigation, drainage, etc.) but total amounts available are constrained by overall regional supplies. Machinery, $i \in M$, is limited by inherited (depreciated capacity) but can be augmented by investment. Finally output balance constraints, $i \in O$, connect the production of commercial crops to the sales activities of the farm. These constraints, say $\mathcal{T} = \mathcal{W} \cup \mathcal{E} \cup \mathcal{M} \cup \mathcal{O}$ together with nonnegative restrictions define the set of technologically feasible activity set T of equation (1) § 2.1.1. Hence,

$$(1') \qquad T(B_T, c_T^t) = \{X^t \,|\, \Sigma_{j\in\mathcal{X}} b_{ij} X_j^t \leqq c_i^t,\ i\in\mathcal{T},\ X_j^t \geqq 0,\ j\in\mathcal{X}\}$$

Household activities involving commercial consumption material and labor purchases and investment activities all compete for working capital. Financial activities involve additions to working capital through borrowing or deductions through debt repayment or cash savings. The former are limited by external banking rules, the latter by borrowing and cash commitments. These involve a set of constraints, $i\in\mathcal{F}$, that determine the set F of financially feasible farm activities of equation (2), § 2.2.1. Hence,

$$(2') \qquad F(B_F, C_F^t) = \{X^t \,|\, \Sigma_{j\in\mathcal{X}}\, b_{ij} X_j^t \leqq C_i^t,\ i\in\mathcal{F}\}$$

We have emphasized the role of learning on the part of farmers in transition and argued that the learning process limits the speed of adoption of new inputs, outputs, or production practices. In a given year a set of adoption constraints, $i\in\mathcal{N}$, limit activities that involve these new things. These constraints define the set L of equation (3), § 2.2.1. Hence,

$$(3') \qquad L(B_L, C_L^t) = \{X^t \,|\, \Sigma_{j\in\mathcal{X}}\, b_{ij} X_j^t \leqq C_i^t,\ i\in\mathcal{N}\}$$

With these definitions we obtain the operational analogue of the theoretical set of feasible regional aggregate decisions

$$(4') \qquad \Gamma(B, C^t) = \{X^t \,|\, \Sigma_{j\in\mathcal{X}}\, b_{ij} X_j^t \leqq C_i^t,\ i\in\mathcal{G},\ X_j^t \geqq 0,\ j\in\mathcal{X}\}.$$

The constraint structure represents at the regional level the household and firm aspects of the farm and the farm's linkages with external sectors. These linkages connect the farm sector to input, output and capital markets. Input markets provide both rural and nonrural variables and quasi-fixed factors of production to the firm, output markets provide an outlet for firm sales and commercial consumption to households, while capital markets provide a source of additional funds to both the farm and household and alternative sources of investing cash savings for the household. In traditional agriculture the important interdependencies also occur between the farm-household and the farm-firm. These include the supply of family labor and cash savings from the household to the firm and the flow of outputs for subsistence consumption from the firm to the household.

3.2. Goal and Satisficing Constraints

3.2.1. Subsistence

Subsistence activity is the result of a combination of physiologically determined needs and socially conditional wants. Various household activities satisfy these needs and wants in varying degrees. We cannot go into an analysis of this complicated and poorly understood transformation here.[16] However, we assume that a well defined utility function exists whose upper contour sets are convex and which can be approximated by a polyhedron defined by a set of hyperplanes. Hence, the set of household activities, H, satisying the subsistence goal can be approximated by the set satisfying the following inequalities

$$\hat{S}(B_S, C_S^t) = \{X^t \mid \Sigma_{j\in H} b_{ij} X_j^t \leqq C_i^t,\ i\in E\} \tag{18}$$

where following the convention established above each of the C_i is negative indicating a lower bound on household activities, and where each $b_{ij} < 0$

indicates an activity that helps meet this lower bound. These describe how satisfaction of anticipated subsistence consumption requirements can be met by planning for adequate amounts of commercial purchases or by using up enough farm produced commodities. In theory the coefficients b_{ij}, c_i, $i \in E$, $j \in H$ depend on the a^{1t} vector and σ_1^t parameter of the utility function φ_1 of equation (5).[17]

If in the course of solving the complete model the subsistence satisficing constraints are satisfied, then explicit maximization of the subsistence goal is unnecessary. If on the contrary no feasible decision in $\hat{\Gamma}(B^t, C^t)$ can be found which also belongs to (18), then an explicit subsistence utility function μ_1 would have to be maximized. In the Punjab during the test period it seems reasonable to suppose that (18) is satisfied at least on the average. Hence we have the approximation

$$(19) \qquad \hat{\Psi}_1^t = S(B_S, C_S^t) \cap \hat{\Gamma}(B_G, C_G^t)$$

3.2.2. Commercial Consumption -- Cash Saving

As we have already noted, commercial consumption competes for working capital, even the smallest cash income of peasant farmers is ordinarily divided between commercial goods and farm inputs. Here we assume the existence of a life cycle utility function μ_2 that preorders choices between current consumption and future anticipated income streams based on the rate of returns to working capital. We have in mind a flexible utility concept KOOPMANS [1964] that achieves a great simplification of simultaneous investment consumption choices and boils down to a consumption function based on cash income and the current internal rate of return. This consumption function represents the satisficing level for cash consumption in the farm region and may be denoted by the function

$$(20) \qquad f(\hat{\rho}^t, Y^{t-1})$$

where f depends in theory on time preference and where Y^t is cash income.[18] The rate of return on saving and the income to be anticipated depends on the profit maximizing solution to the production and investment decision problem. The determination of all of these variables would be too complicated for a farmer to solve simultaneously. Instead we assume in the operational model that $\hat{\rho}_t$ is estimated from recent farm profit experience and the current bank rate. In this way planned cash consumption X_H^t and planned cash savings $Y^t - X_H^t$ are determined. The latter then augments the supply of working capital so that X_H^t enters the appropriate financial constraint defining F above. We also assume that cash consumption is partially allocated to alleviating subsistence needs by substituting costly purchased foods for inexpensive home produce so that this variable also enters (18). The set of activities satisfying the consumption goal (20) may be denoted

$$(21) \qquad H(B_H, C_H^t) = \{X^t \,|\, X_H^t \geqq C_H^t\}$$

Hence we have the approximation

$$(22) \qquad \hat{\Psi}_2^t = H(B_H, C_H^t) \cap \Psi_1^t$$

where $\hat{\Psi}_1^t$ is given by (19).

3.2.3. Safety

The safety metric can be introduced as a fundamental axiom of behavior DAY [1971], it can be derived from the safety-first ROY [1952] or focus-loss SHACKLE [1958] principles. In any of these cases the safety metric circumscribes decisions by an ellipsoidal "Safety Zone" which can be approximated by supporting hyperplanes as in the case of the subsistence goal.[19] In this study we represent the safety-zone by three sets of linear inequalities. The first two are sets of upper and lower bounds on crop acreages which prevent extreme changes in cropping patterns and protects the farmer against a drastically changed pattern of relative profitabilities in commodities at the end of the season. The second prevents extreme increases in capital stocks and protects the farmer from sinking too much capital in one opportunity when another, perhaps currently unknown one, may be more desirable in the future. As we shall see below these several bounds are adaptive in form.

A set of upper and lower bounds on individual cash crop acreages represent the first two sets of constraints: the upper bounds are

(23) $\quad \Sigma_{j \epsilon P_i} \; X_j^t \leqq C_{i'}^t \geqq 0,\; i \epsilon S,\; i' \epsilon S^u$, where to each $i \epsilon S$ there corresponds exactly one $i' \epsilon S^u$.

and the lower bounds

(24) $\quad \Sigma_{j \epsilon P_i} X_j^t \leqq C_i^t \leqq 0,\; i \epsilon S,\; i'' \epsilon S^{\ell}$, where to each $i \epsilon S$ there corresponds exactly one $i'' \epsilon S^{\ell}$.

The constraints in the third group have indexes $i' \epsilon M'$, one for each machine or draft-power investment activity, $j \epsilon J_m$. These bounds are

(25) $\quad X_{j_i}^t \leqq C_{i'}^t,\; j_i \epsilon I_m,\; i' \epsilon M',\; i \epsilon M$.

in which to each $i \epsilon M$ there corresponds exactly one $i' \epsilon M'$.

The set of all safety factors is $Z = \mathcal{S}_h^{\mu} \cup \mathcal{S}_h^{\ell} \cup \mathcal{M}'$; the safety zone Z within which safety requirements are satisfied is the set of decisions

(26) $$\hat{Z}(B_Z, C_Z^t) = \{X^t | \Sigma_{j \in X} b_{ij} X_j^t \leqq C_i^t,\ i \in Z\}$$

We now have the approximation

(27) $$\hat{\Psi}_3^t = Z(B_Z, C_Z^t) \cap \hat{\Psi}_2^t$$

where $\hat{\Psi}_2^t$ is approximated by (22). However, it is possible under some circumstances of price, income and technology that the safety constraints cannot be satisfied while at the same time meeting subsistence and cash consumption requirements. If this is the case the utility function μ_3 measuring the distance from Z to the set $\hat{\Psi}_2$ would have to be minimized. This would mean that subsistence would require a more daring departure in production patterns than the farmers' currently desire. As before we suppose that $\hat{\Psi}_3^t$ is non-empty in the Punjab during the test period.

3.2.4. Profits

The anticipated costs and returns of particular activities that enter the profit objective function φ_4 fall into four classes: expected returns from sales activities $a_j^t \geqq 0$, $j \in \mathcal{S}$, current costs of purchasing activities $a_j^t \leqq 0$, $j \in \mathcal{B}$, an annual depreciation charge $a_j^t \leqq 0$, $j \in \mathcal{I}_m$ that must be recovered to justify an investment, and interest costs and returns associated with borrowing activities. If $\hat{p}_i^t$ is the expected unit price of crop i then $a_{j_i}^t = \hat{p}_i^t$, $j_i \in \mathcal{S}$, $i \in \mathcal{O}$. Likewise, if p_i^t is the current price of inputs then $a_{j_i}^t = p_i^t$, $j_i \in \mathcal{B}$, $i \in \mathcal{E}$. In the case of investment goods the depreciation charge based on straight line depreciation, is the current investment good price p_i^t divided by the average life λ_i, $i \in \mathcal{M}$. Hence, $a_{j_i}^t = -p_i^t/\lambda_i, j_i \in \mathcal{I}, i \in \mathcal{M}$.

In the case of borrowing a_j is equal to the negative of the interest rate in that category of loans. For saving it is the positive average bank rate. In order to account for strong liquidity preference we include a transfer activity of working capital to the farm "investment account" at a cost determined by an internal risk premium. This premium is computed so that farm investment will occur only if its pay back period is five years or less. All other a_j coefficients are zero.

With these several assumptions the profit objective corresponding to φ_4 becomes

$$< X^t, a^t > = \Sigma_{j\in X} a_j^t X_j^t \tag{28}$$

3.3. The Approximating Linear Program

The aggregate of farm decisions in the region are now represented for each year by the linear programming problem: maximize the profit function (28) subject to all of the technical, financial, learning and satisficing constraints. Let $\overline{B}^t$ be the matrix of b_{ij} coefficients and $\overline{C}^t$ the vector of C_j coefficients entering Γ and S, C and Z and let

$$\Lambda(\overline{B}^t, \overline{C}^t) = \Gamma(B^t, C^t) \cap C(B_c^t, C_c^t) \cap H[B_h^t, C_h^t] \cap Z[B_z^t, C_z^t] \ . \tag{29}$$

The set of solutions $\hat{\Psi}_4^t$ to the linear program

$$\Pi^t = \max_{x^t} \{< x^t, a^{4t} > \mid x^t \epsilon \Lambda(\overline{B}^t, \overline{C}^t)\} \tag{30}$$

approximates Ψ_4 or equivalently Ψ of equations (8) and (9) under the assumption that all the high order subsistence, consumption, saving and safety goals can be satisfied. We thus reduce the L* programming problem to an ordinary linear programming problem with the technical constraints augmented by constraints representing high order goal fulfillment.

3.3. Feedback Functions and Exogenous Variables

The profit coefficient vector a^t, the constraint coefficient matrix $\bar{B}_t$, and the limitation coefficient vector $\bar{C}^t$ which determine regional behavior according to the approximating linear program (30) must be estimated for each production period. In some cases we have treated these data as exogenous, delaying an analysis of them to model refinements in the future. In other cases we have adopted specific behavioral hypotheses about how these anticipated payoffs and constraining factors are adaptively formed by the farmers. We shall now present an outline of our specific assumptions.

3.3.1. Labor and Materials

Variable inputs including labor and materials are (except for family labor) assumed not to be inventoried on the farm so that needed supplies must be purchased by the appropriate purchasing activity in the set $\mathcal{B}$. The materials constraints, $i \in \mathcal{E}$, are divided into two groups, a set of balance constraints representing the purchase requirements placed by farm demand and a set of purchasing restrictions that limit purchases to exogenously given supplies. Let $\mathcal{E}_1$ be the first group and $\mathcal{E}_2$ be the second. Then $C_i^t = 0$, $i \in \mathcal{E}_1$ and $C_i^t = Z_i^t$, $i \in \mathcal{E}_2$ is the exogenously given supply of input $\bar{C}_i$. In some cases we have assumed that C_i^t is infinite, such as electricity and regional labor. In others we estimated a finite magnitude such as family and village labor.

3.3.2. Capital Goods

Land is assumed to be nondepreciating so that

$$(31) \qquad C_i^t = C_i^{t-1} - b_{ij_i} x_{j_i}^{*t-1}, \; i \in L, \; j_i \in \mathcal{I}_\ell$$

where $\mathcal{I}_\ell$ is a set of investment activities in land. (We recall that $b_{ij} < 0$ implies an output). For machinery and draft power we assume a "one-hoss shay" process

$$C_i^t = \sum_{s=1}^{\lambda_i} - b_{i_{j_i}} X_{ji}^{*t-s}, \ i \epsilon M, \ j_i \epsilon I_m \ . \tag{32}$$

in which λ_i is the use life of machine $i \epsilon M$.

3.3.3. Financial Constraints

Four financial constraints are included in the current model. The first specifies that working capital expended on consumption, or transferred to the farm account for purchases and investment, cash savings and debt payment cannot exceed the amount available, which depends on past sales, augmented by current borrowing. The second requires that all debts be repaid at the beginning of the year so that longer term borrowing is considered to be a series of short term loans.[20] A third specifies that borrowing is limited to a portion of current "quick assets" an assumption assumed to represent loan practices of bankers. The fourth, is a balance constraint limiting the transfer of working capital to the investment account from the available supply.

With these assumptions we get a set of constraint that depend on past sales, and financial activities; hence we have in summary

$$\begin{aligned}
C_{f_1}^t &= \Sigma_{j \epsilon S} \, a_j^{t-1} \, X_j^{*t-1} \\
C_{f_2}^t &= \Sigma_{j \epsilon F_b} \, X_j^{*t-1} \qquad , \text{ where } f_i \epsilon F \\
C_{f_3}^t &= \Sigma_{j \epsilon S} \, a_j^{t-1} \, X_j^{*t-1} \\
C_{f_4}^t &= 0.
\end{aligned} \tag{33}$$

3.3.4. Adoption

Learning new technology is partially based on exposure and which can be measured by the "amount" already adopted. Specifically, we assume that exposure is proportional to use, and that use is measured by the total activity level X_j^{t-1} $j \in N$, already allocated to the new activity. We then get

$$(34) \qquad C_i^t = (1 + a_{j_i}) X_{j_i}^{t-1}, \; i \in N, \; j_i \in X$$

This, it must be remembered, gives the maximum expected amount of adoption in the region under conditions favorable to it. If it is currently uneconomic, or if other constraints prevent it, adoption in a given year will fall below this amount. The model then explains internally whether or not adoption will proceed according to this maximal rate.[21]

3.3.5. Subsistence Satisficing Constraints

The subsistence requirements are determined exogenously by the number of farm families and by survey data on home consumption.[22]

3.3.6. Commercial Consumption

If we let ρ^{t-1} be the lagged internal rate of return on working capital and i^t be the current exogenously given bank rate, and if we let Y^{t-1} be the lagged sales which depend on X_j^{*t-1}, $j \in S$ then

$$(35) \qquad C_H = f[\max \{i, \rho^{*t-1}\}, \Sigma_{j \in S} \, a_j^{t-1} X_j^{*t-1}] \; .$$

3.3.7. Safety

The first set of safety limitation coefficients have the adaptive form

$$(36) \qquad C_{i'}^t = (1 + \beta_i^u) \, \Sigma_{j \in P_i} X_j^{*t-1}, \; i \in S, \; i' \in S^u$$

$$(37) \qquad C_{i''}^t = -(1 - \beta_i^{\ell}) \, \Sigma_{j \in P_i} X_j^{*t-1}, \; i \in S, \; i'' \in S^{\ell}$$

in which P_i in the set of production activities using land to produce commercial crop $i \in S$. The constraints corresponding to (36)-(37) are called flexibility constraints because they describe how flexible a farmer is in any one year in modifying his cropping patterns to take advantage of currently profitable opportunities.[23]

The second set of safety constraints are based on the old idea that capital stock is adjusted more or less gradually because of the risks involved in immediate adjustment. If C_i^t, $i \in M$ is the amount of capital service available in the region in year t of the i^{th} capital good and $\overline{C}_i^t$ the maximum amount that could be used under any condition, then the current maximum investment potential is $\overline{C}_i^t - C_i^{t-1}$, $i \in M$. The adjustment limitation is then

$$(38) \qquad C_{i'}^t = \gamma_i [\overline{C}_i^t - C_i^{t-1}], \; i' \in M', \; i \in M$$

where γ_j is an adjustment coefficient and to each $i \in M$ there corresponds exactly one $i' \in M'$. Because of the depreciation relation (32) we have

$$(38') \qquad C_{i'}^t = \gamma_{i'} [\overline{C}_{i'}^t + \sum_{s=1}^{\lambda_i} b_{i_{j_i}} X_{j_i}^{*t-1}] \; i \in M, \; i' \in M'$$

These bounds, let it be emphasized, are upper bounds and will be reached only if investment appears to be profitable and if other factors such as learning, financing, labor, etc., are not limiting.[24]

3.4. The RLP Model

The feedback functions (31)-(38) provide an operational approximation of equation (16). The linear programming problem whose algorithmically selected solution approximates (15) is given by (30). The operational model then consists of a sequence of linear programming problems each one of which is used to estimate production, household, investment and marketing activities in the region for a given year, and the feedback functions which represent how the region's external environment influences farmers' decision problem, how new information is incorporated and how behavioral parameters are adaptively modified on the basis of experience and new conditions.[25]

The imperfections in this operational model are evident and no doubt numerous improvements can and one day should be made. At this point, however, we shall concentrate on a detailed evaluation of the model's ability to track recent history. Our objective is to find out if it can be used -- in its present form -- for projections and policy analysis.

4. ESTIMATION AND EVALUATION

4.1. Estimation[26]

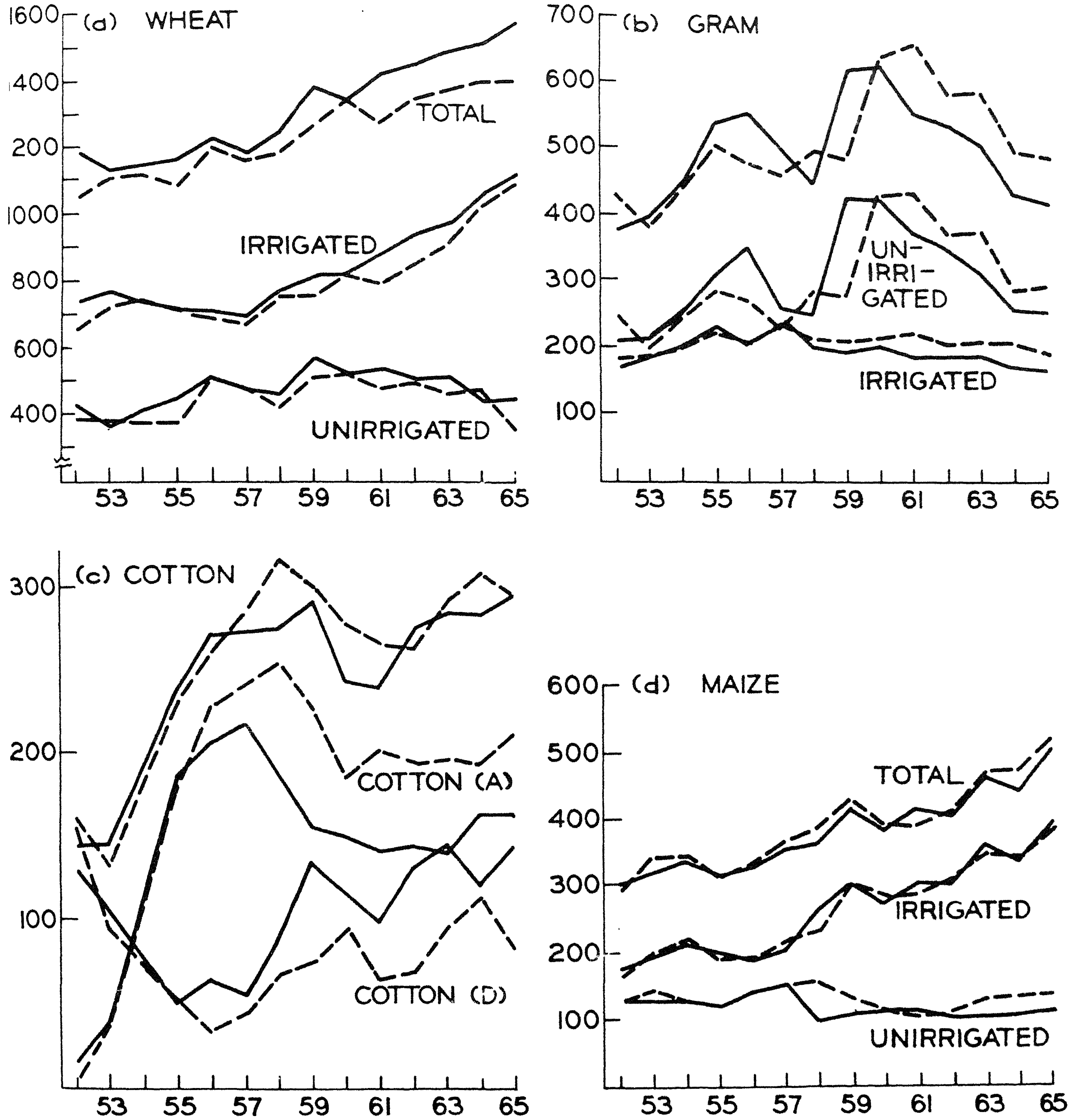
The operational farm sector model has been estimated for the central districts of the Indian Punjab. The b_{ij} coefficients for farm technology are obtained from sample survey data while crop yield coefficients were derived from discrete approximations to continuous production functions fitted to data available from field trials. The prices for both variable and capital inputs and the prices for farm outputs were obtained from regional statistical abstracts. The regional availability of physical resources were obtained from national and state census data, while coefficients appearing in the feedback function (34) - (38) were partly estimated using regression analysis. In some cases, where no "objective" statistical method was possible, the estimates of regional experts were used.[27] A complete exegesis of the data and procedures employed for estimation is reported elsewhere (SINGH [1971]).

4.2. Model Results and Evaluation

The model was used to simulate regional agricultural history for the period 1952-1965. The results include a set of variables for which comparable regional data exist. In this set are the acreages sown to various crops over the 14 year period. They also include variables for which no comparable data are available, such as predicted levels of resource use for family labor, hired labor, animal draft and various machine capacities, levels of investments and capacity used of new power sources, levels of production, sales (marketed surplus) and retained consumption of various farm outputs, use of chemical fertilizers by crop and predicted levels of grain sales, working capital used, borrowings at various rates of interest and savings, all on a regional basis. The first set provides the basis for our model evaluation.[28] It if provides evidence that the model does capture a significant part of reality then variables in the second set may be regarded as useful new estimates of economic activity during the period. Moreover, the model would in that case presumably be useful for projecting likely future developments in the region under current and alternative policy programs. In the paper we concentrate on a comparison of the model generated data with comparable "observed" series. A complete description of all the model results is contained elsewhere.

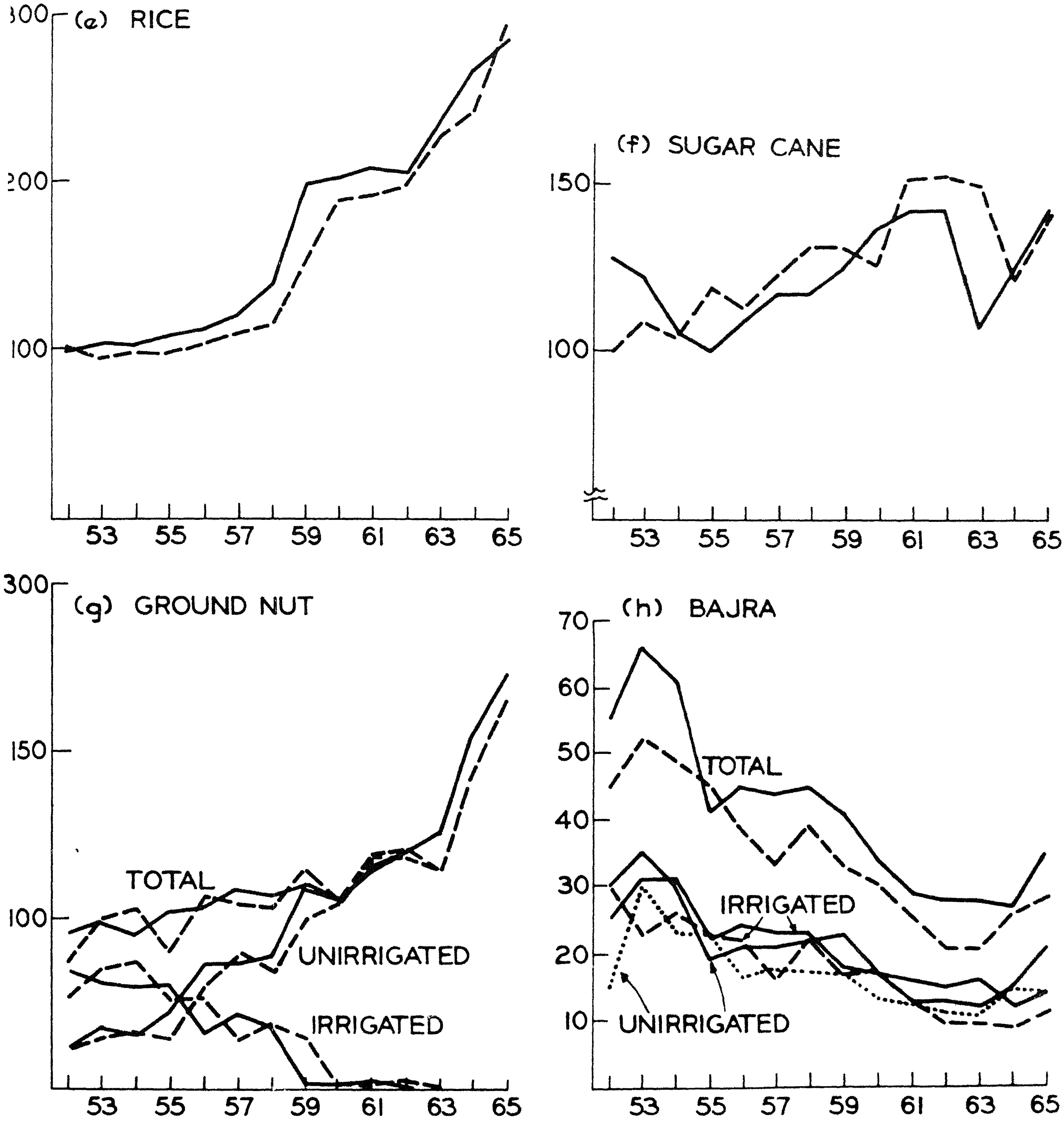
Ideally we should like to compare each model estimate $X_j^{*,t}$ with its realized counterpart $X_j^{o,t}$. This is not possible because data do not exist in sufficient detail. However, various observed aggregate series can be compared with the corresponding model aggregates. Let P_i^t, $i = 1,\ldots,q$ be an aggregate variable available for year t. Let P_i^{ot} stand for the "observed" data and P_i^{mt} stand for the corresponding variable for the model obtained by aggregating for period t the appropriate regional activity levels X_j^{*t}. We then have two series: P_i^{ot}, $t = 1952,\ldots,1965$, P_i^{mt}, $t = 1952,\ldots,1965$, that may serve as the basis for a model evaluation.

The specific aggregate series available include irrigated, unirrigated and total crop acreages for the nine major crops in the central Indian Punjab. These include the winter (Rabi) crops wheat, gram and barley, and the summer (Kharif) crops cotton, maize, rice, groundnut and bajra (spiked millets), and an annual sugarcane crop that spans both cropping seasons. The several series are displayed graphically in Figure 1 (except for barley whose acreage is insignificant). These provide a visual impression of model performance to be supplemented by the statistical measures to follow.



——— OBSERVED ----- MODEL EXPLANATION

FIG. 1: OBSERVED AND MODEL EXPLANATIONS OF FIELD CROP ACREAGE IN THE CENTRAL PUNJAB, 1952-1965 (THOUSANDS OF ACRES)



——— OBSERVED   - - - - - MODEL EXPLANATION

FIG. 1: CONTINUED (THOUSANDS OF ACRES)

Various statistical methods can be used to evaluate model performance. We have chosen several that allow various specific characteristics of the model generated and the observed series to be analyzed. The characteristics we consider are (1) absolute levels, (2) magnitudes of change, (3) relative variable levels, (4) directions of change and (5) turning points. The ability of the model to "explain" these characteristics in the observed data is compared with a naive model appropriately defined in each case. The first two are discussed in the next section. This is followed by a discussion of the next three, all of which rely on information theoretic concepts.

4.3. Evaluation I: Variable Levels and Magnitudes of Change

4.3.1. Absolute Variable Levels

Based on a suggestion of J.J. Johnson, COHEN and CYERT [1961] recommend evaluating a simulation model's ability to explain the absolute levels of an observed series by regressing each observed series on the corresponding model series under the assumption that the observed series contains a systematic part represented by the model and a random error. The evaluation involves estimating the regression

$$P_i^{ot} = \alpha_i + \beta_i P_i^{mt} + \epsilon_{it}, \quad i = 1,\ldots,q, \quad t = 1,\ldots,T \tag{39}$$

and testing the hypotheses $H_{oi}: \beta_i = 0$, $H_{\alpha i}: \alpha_i = 0$ and $H_{\beta_i}: \beta_i = 1$. If H_{oi}: is rejected for a given i then the model is thought to capture a significant part of the variation in absolute levels of the variable involved. If H_{α_i} or H_{β_i} is rejected for any i then the model is thought to produce significantly biased estimates of the levels of that variable.

The results of applying this procedure are displayed in Table 1.

TABLE 1: REGRESSION EVALUATION OF MODEL EXPLANATION OF OBSERVED LEVELS OF FIELD CROP ACREAGES.

Statistic / Crop	$\hat{\alpha}$	t_{α}	$\hat{\beta}$	t_{β}	R^2
Wheat (T)	71.27	0.6054	0.9960	0.0427	.9045
Wheat (I)	25.79	0.4339	1.018	0.2447	.9413
Wheat (U)	100.36	1.4015	0.8354	1.0459	.7012
Gram (T)	168.29	1.5021	0.6447	1.639	.4343
Gram (I)	-12.74	0.2215	0.9960	0.0145	.5219
Gram (U)	101.19	1.5538	0.6666	1.591	.4574
Barley (U)	24.66	6.0049	-.23	5.309	.0759
Cotton (T)	31.02	1.5356	0.8403	2.069	.9081
Cotton (I)	48.35	2.5113	0.6689	1.458	.4197
Cotton (A)	17.028	1.126	0.7227	3.486	.8731
Maize (T)	30.30	1.3384	0.8953	1.8388	.9537
Maize (I)	-3.505	0.2543	1.0233	0.4576	.9711
Maize (U)	73.72	2.0218	0.3305	2.4046	.1051
Rice (I)	13.92	1.3266	0.9973	0.0447	.9564
Sugarcane (T)	73.93	2.7936	0.3867	2.9722	.2264
Groundnut (T)	-0.94	0.7607	1.0533	0.5672	.9129
Groundnut (I)	-0.60	0.012	0.914	0.6878	.8168
Groundnut (U)	3.028	0.5721	1.0548	1.1804	.9773
Bajra (T)	1.734	0.3848	1.1323	1.0676	.8743
Bajra (I)	6.138	1.4397	0.8302	0.7171	.5058
Bajra (U)	6.397	1.8282	0.8166	0.9875	.6171

T = Total; I = Irrigated; U = Unirrigated

A glance at the column R^2 gives a good idea of the closeness of fit for individual crops. The model predicts the acreage levels very well for most crops -- wheat (total and irrigated), cotton (total and American), maize (total and irrigated), rice, groundnut (total, irrigated and unirrigated) and bajra (total); moderately well for two -- wheat (unirrigated) and bajra (unirrigated); and very poorly for barley (unirrigated), maize (unirrigated) and sugarcane. The results for these three crops are poor in all respects.

The "t" values indicate that the $\hat{\alpha}$ estimates are different from zero only for barley (unirrigated), cotton (D) and sugarcane, and for maize (unirrigated) at the 5% level of significance. The "t" values indicate that the $\hat{\beta}$ estimates are significantly different from unity for cotton (American), maize (unirrigated), sugarcane, maize (unirrigated) and barley (unirrigated).[29]

Serious objections to the use of the above analysis can be raised: the model estimates are not independent while the tests assume they are; and, the test takes no account of the relative importance of the several variables tested. The first objection vitiates the theory of significance lying behind the t ratios. Hence, at best the statistics of Table 1 must be regarded as <u>informal</u> measures of goodness of fit and model bias, that tend to overestimate model error. Nonetheless they are effective in a descriptive way, and on the basis of them we gain the impression that the Punjab model is fairly effective at estimating field crop levels, though not with great precision.

4.3.2. Magnitudes of Change

The magnitudes of change predicted by the model and those observed are displayed in the prediction-realization diagrams shown in Figure 2. These diagrams show that while the model correctly predicts the direction of change more often than not, it predicts the levels of change with no great accuracy. A useful measure of how close to the observed magnitudes of change these model generated magnitudes are is found in the "inequality coefficient" THEIL [1966] which, using our notation, is defined to be

$$(40) \qquad U_i = \{\Sigma_{i=1}^{n} [\Delta P_i^{mt}/P_i^{mt} - \Delta P_i^{ot}/P_i^{ot}]/\Sigma_{i=1}^{n} \Delta P_i^{ot}/P_i^{ot}\}^{1/2}.$$

If percentage changes are perfectly predicted for a given crop i, then $U_i = 0$. A naive model that predicts no change in a variable would yield an inequality statistic of 1. Hence values of U_i between 0 and 1 indicate a model performance better than a no change prediction; the statistic has no finite upper limit because it is possible to make a prediction worse than a no change prediction. Table 3 gives the statistic for the aggregate field crop series.

TABLE 2: INEQUALITY COEFFICIENTS FOR THE PUNJAB MODEL

Crop	Inequality Coefficient
Wheat	1.2192
Gram	1.1703
Cotton	.6879
Sugarcane	1.4407
Maize	.6513
Groundnut	1.4413
Rice	.6481
Bajra	.9896

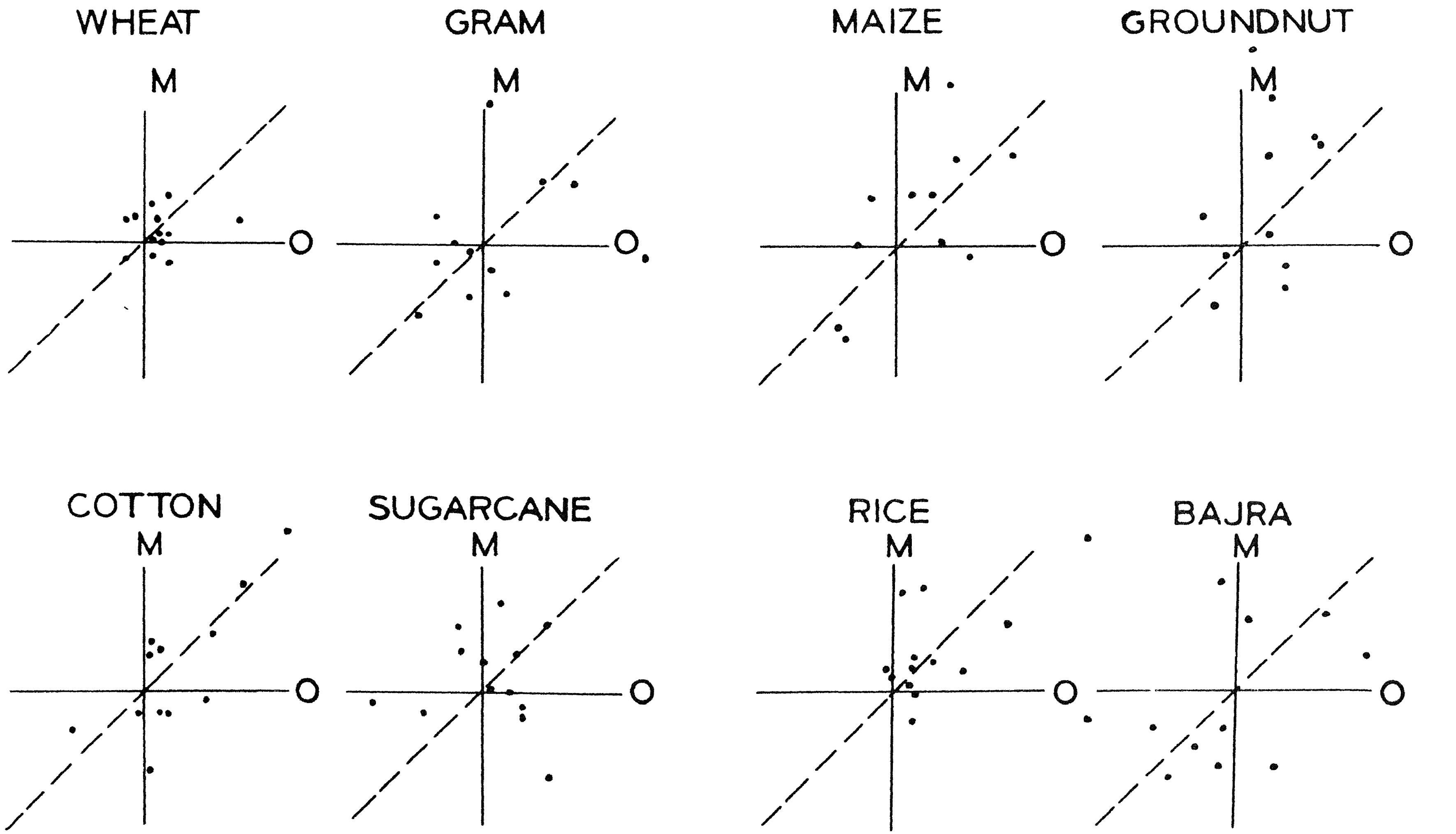


FIG. 2: PREDICTION-REALIZATION DIAGRAMS FOR TOTAL CROP ACREAGES

M-AXIS MEASURES PERCENTAGE ACREAGE CHANGES "PREDICTED" BY THE MODEL
O-AXIS MEASURES ACREAGE CHANGES "OBSERVED"

The model performance "beats" the naive alternative in half the cases: cotton, maize, rice and bajra. In interpreting these results it should again be borne in mind that the crop estimates are interdependent, while the statistic is based on the assumption of crop estimate independence. The results are biased and overemphasize model error. Nonetheless, our conclusion must be that the model is a generally inaccurate predictor of annual percentage changes in field crop levels. The fact, however, that it usually predicts the direction of change correctly (in a total of 70% of the cases) leads us to investigate its ability to capture qualitative variation in the data. We can take this up later on the basis of information theoretic concepts to which we now turn, first however using them to evaluate the model's ability to explain the allocation of land in relative terms.

4.4. Evaluation II: Information Statistics

4.4.1. Information

We turn now to a series of tests based on the concept of statistical information introduced into econometric work and expounded extensively by THEIL [1967]. Let p be a variable belonging to the interval [0, 1]. Its information content is defined to be

$$(41) \qquad h(p) = \log (1/p) = -\log (p) = -h\ (1/p)$$

and its expected information content is

$$(42) \qquad ph(p)$$

where the reasonable convention $ph(p) = 0$ if $p = 0$ is assumed. If $p_i, i = 0,\dots,n$ are a set of variables such that $p_i \epsilon [0,1]$ and $\Sigma_i^n p_i = 0$ then the expected information content of these variables is defined to be

$$(43) \qquad H(p) = \Sigma_{i=1}^{n} p_i h(p_i) \ .$$

4.4.2. Relative Magnitudes

We use (43) first to evalute the model's ability to explain the proportion of land allocated to various field crops, an application analogous to that of TILANUS and THEIL [1965]. In this application we define

$$(44) \qquad p_i^{mt} = P_i^{mt}/\Sigma\ P_i^{mt}, \quad p_i^{ot} = P_i^{ot}/\Sigma\ P_i^{ot}$$

where the summation is over either all of the field crops or over the crops by season. We shall indicate which in each case. The expected information content in the observations for each year is now

$$(45) \qquad H(p^{ot}) = \Sigma_{i=1}^{n} p_i^{ot}\ h(p_i^{ot})$$

and the average is

$$(46) \qquad \overline{H}(p^{o}) = \frac{1}{T}\ \Sigma_{t=1}^{T}\ H(p^{ot})$$

The <u>information inaccuracy</u> of the model is

$$(47) \qquad I(p^{ot},\ p^{nt}) = \Sigma_{i=1}^{n}\ p_i^{ot}[h(p_i^{ot}) - h(p_i^{mt})]$$

a statistic that is portional to "the weighted variance of the relative prediction errors ... so that errors in less important coefficients are weighted less heavily than the same relative errors in larger coefficients." [TILANUS and THEIL op.cit. p. 850.]

The average information inaccuracy is

$$(48) \qquad \overline{I}(p^{o},p^{m}) = \frac{1}{T}\ \Sigma_t\ I(p^{ot},p^{mt})$$

The relative information inaccuracy is now defined to be

$$(49) \qquad R(p^{ot},p^{nt}) = I(p^{ot},p^{nt})/H(p^{ot})$$

an indication of what proportion of the total information content of the observation has been lost in a given year. The average of this figure is

$$(50) \qquad \bar{R}(p^{o},p^{m}) = \Sigma_t \; R(p^{ot},p^{mt}) \; .$$

The expected information content of the observations for all crops is shown in column (3) of Table 4 for each year. The information inaccuracy is shown in column 2 while the relative information inaccuracy is shown in column 4. Although no level of significance can be assigned to these non-parametric statistics, it is obvious that the model predicts the proportions extremely well since in no year does the relative information loss in the model exceed 0.6 percent, while on the average the model loses less than 0.3 percent of the information contained in the observed proportions.

These results may be compared to those obtained from a naive model that predicts a variable $p_i^{nt} = p_i^{o,t-1}$. To facilitate this comparison we have computed the ratios of the relative information inaccuracy statistics for the two models separately for Rabi and Kharif crops:

$$(51) \qquad R(p^{ot},p^{mt})/R(p^{ot},p^{nt}) = I(p^{ot},p^{mt})/I(p^{ot},p^{nt}).$$

This relative measure is shown in columns 5 and 6 of Table 4. These ratios vary from year to year but on the average indicate that the model explanation reduces the information inaccuracy 1 1/2 times for the rabi season and nearly 8 times for the kharif season over the information loss in the naive model.

TABLE 3: INFORMATION STATISTICS FOR EVALUATING MODEL PERFORMANCE ON RELATIVE MAGNITUDES.

	ALL FIELD CROPS			RABI CROPS	KHARIF CROPS
(1) YEAR	(2) INFORMATION INACCURACY	(3) EXPECTED INFORMATION CONTENT	(4) RELATIVE INFORMATION INACCURACY	(5) RATIO OF RELATIVE INFORMATION INACCURACY MODEL÷NAIVE	(6) RATIO OF RELATIVE INFORMATION INACCURACY MODEL÷NAIVE
1952	.006682	1.620	.004123	N.A.	N.A.
1953	.001771	1.644	.001077	0.2606	3.0346
1954	.001682	1.648	.001021	0.0832	0.3666
1955	.002866	1.642	.001758	0.1255	0.2149
1956	.003037	1.651	.001839	0.2214	0.0352
1957	.005032	1.699	.003016	0.4414	0.0936
1958	.006215	1.664	.003735	1.2407	0.2079
1959	.00968	1.680	.005763	0.6049	0.1709
1960	.001575	1.665	.000946	0.0545	0.0885
1961	.006586	1.655	.00398	1.3389	0.0823
1962	.003396	1.657	.002049	1.2023	0.0374
1963	.008084	1.661	.004867	1.7161	0.1418
1964	.002694	1.665	.001618	0.2697	0.0475
1965	.002505	1.688	.001484	0.4852	0.0202
Average	.003794	-	.002406	0.6484	0.1255

4.4.3. Qualitative Model Performance

The qualitative performance of the model with respect to directions of change displayed graphically in Figure 3 can be summarized conveniently in a "prediction-realization", or an explanation-observation table as follows.

TABLE 4: EXPLANATION-OBSERVATION TABLE

		OBSERVED			
		INCREASE	NO CHANGE	DECREASE	ROW SUM
EXPLAINED	INCREASE	f_{11}	f_{12}	f_{13}	$f_{1.}$
	NO CHANGE	f_{21}	f_{22}	f_{23}	$f_{2.}$
	DECREASE	f_{31}	f_{32}	f_{33}	$f_{3.}$
	COLUMN SUM	$f_{.1}$	$f_{.2}$	$f_{.3}$	1

The table gives the relative frequency f_{ij} of a type i change "predicted" or explained by the model when a type j change was "realized" or observed in an aggregate time series (i,j = 1,2,3 for increase, no change and decrease in the levels respectively). The observed frequency of a type i change is given by $f_{.i}$ and the explained frequency by $f_{i.}$. True predictions are given by the frequencies f_{ii} along the diagonals. We can easily calculate the relative frequency of correct predictions ($\sum_i f_{ii}$), and of various kinds of errors such as underestimation of change ($f_{21} + f_{22}$), overestimation of change ($f_{12} + f_{32}$) and turning point errors ($f_{13} + f_{31}$).

Using the expected information concept of equation (43) we now define the <u>observed information</u> with respects to the events enumerated in the explanation table to be

(52) $$I_o = \Sigma_i\, f_{.i}\, h(f_{.i})\ .$$

It is this qualitative information that we wish to explain with the model. Because

(53) $$\Sigma_i f_{.i} h(f_{.i}) = \Sigma_i f_{ii} h(f_{.i}) + \Sigma_i [\Sigma_{i \neq j} f_{ij} h(f_{.i})]$$

I_o can be decomposed into the <u>explained information</u>

(54) $$I_o^T = \Sigma_i f_{ii} h(f_{.i})$$

representing the observed information correctly explained by the model and the <u>unexplained information</u>

(55) $$I_o^F = \Sigma_i [\Sigma_{i \neq j} (f_{ji}\ h(f_{.i})] = [f_{.i} - f_{ii}] h(f_{.i})$$

representing the observed information incorrectly explained by the model. The explained information relative to the total content of the observations is then

(56) $$I_o^T / I_o$$

The above measures of qualitative model performance can be computed for any alternative model. We have computed them for the RLP model and for a naive model. For convenience we define

(57) $$I^{\beta}(\gamma)$$

to be the information statistic of character β (explained or unexplained) computed for model γ (RLP or naive). We apply the above concepts to measure the model's ability to capture two kinds of qualitative information, (1) direction of annual change and (2) changes in direction of change or correct turning points.

4.4.4. Directions of Change

In the case of directions of change the f_{ij} are the relative frequences of increases (i,j = 1), no change (i,j = 2) and decreases (i,j = 3) in the levels of field crops from year to year. In Table 5 we present the information statistics for both our Punjab RLP model and for a naive alternative in which the direction of change is predicted to be the same as it was in the year preceding.

In general the RLP model outperforms the naive alternative. It correctly explains roughly 50% more of the observed information. On an individual crop basis it is better in both seasons and it beats the naive alternative for six of nine crops, ties on one (rice) and does less well on two (gram and sugarcane).

4.4.5. Correct Turning Points

Directions of change are only one qualitative characteristic of a time series. Turning points give us another important characteristic, and we now apply the information concept to evaluate the model's ability to successfully explain these changes or lacks of change in the directions of change.

Table 6 presents the results. The model generally outperforms the naive alternative explaining 38% of the observed information in all turning points about 10% better than the naive explanation. By season the predictions are better for Kharif than for Rabi crops and on a crop by crop basis the RLP model beats the naive model four of nine times.

4.5. Summary of the Model Evaluation

Enough evidence has now been accumulated to obtain a good impression of how well our RLP model captures reality at least so far as recent history

TABLE 5: INFORMATION STATISTICS FOR

DIRECTIONS OF CHANGE AND TURNING POINTS

CROP	DIRECTIONS OF CHANGE	
	RELATIVE EXPECTED INFORMATION RLP MODEL	RELATIVE EXPECTED INFORMATION RLP/NAIVE
ALL CROPS	.58	1.49
RABI CROPS	.57	1.18
KHARIF CROPS	.59	1.68
WHEAT	.51	1.94
GRAM	.53	.69
BARLEY	.53	2.38
COTTON	.60	1.43
MAIZE	.66	2.88
RICE	.27	1.00
SUGARCANE	.33	.87
GROUNDNUT	.68	3.03
BAJRA	.85	4.89

CROP	TURNING POINTS	
	RELATIVE EXPECTED INFORMATION RLP MODEL	RELATIVE EXPECTED INFORMATION RLP/NAIVE
ALL CROPS	.38	1.10
RABI CROPS	.26	.79
KHARIF CROPS	.41	1.24
WHEAT	.46	1.41
GRAM	.07	.22
BARLEY	.15	.47
COTTON	.16	.34
MAIZE	.58	1.75
RICE	.27	.88
SUGARCANE	.39	.81
GROUNDNUT	.44	1.33
BAJRA	.61	1.83

in the Punjab goes. No definitive <u>conclusions</u> can be drawn from this evidence, primarily because the statistical measures used are not fully understood from a theoretical point of view. Instead we present as our own <u>opinions</u> the following general impressions: (1) the model fairly accurately explains levels and magnitudes of change of field crop acreages; (2) it explains extremely well the pattern of cropping in the region from year to year; and (3) it explains directions of change and turning points with some -- perhaps surprisingly great -- accuracy.

We believe the evidence supports the inference that the RLP model captures a significant part of the structure of the agricultural economy of the Punjab; that it supports the theory of farm decision making presented above; and while scarcely an accurate predictor of annual events and while clearly leaving plenty of room for possible improvements, it is good enough to use now both for gaining a clearer understanding of past development and for projecting likely future developments under presently conceived policy alternatives. We shall report our applications for both these purposes in another place.

5. POTENTIAL APPLICATIONS

We began our discussion by calling attention to the strategic details of development, an analysis of which is needed if we are to obtain a complete understanding of economic change and if we are going to provide assistance for development policy that relates directly to the microeconomic level where it must be worked out. We developed an abstract theory and a specific mathematical model of decision-making and technology within the farm sector for testing the theory. Finally, we have provided evidence that the model "works," that it is indeed capable of simulating past developments, and that it is therefore more or less realistic. We infer from this that the model can be used for projecting probable future trends for various assumed values of the exogenous variables including those that are policy instruments of government planners. Let us consider some of the possibilities.

5.1. Projections and Early Warnings

By first extrapolating trends in exogenous variables the model can be used to project the values of all the endogenous variables. We are currently in the midst of such model projections to 1980. From these computations we hope to get some idea about the aggregate supply of food commodities, the effect of mechanization on labor utilization in the region, the demand for nonfarm inputs and so on. Far from being a crystal ball it is still possible for the model to serve as a rough guide to the future and perhaps as a useful early warning system. Is it possible, to cite one question as an example, that farm sector development will eventually push farm workers from the sector as it has already so effectively done in Europe and the U.S.? If so, advance warning of the magnitude of the prospective flow of peoples -- even of a rough order -- may stimulate policy to retard the flow, or prepare for its consequences.

5.2. Comparative Dynamics

Comparative dynamics involves simulating the model under alternative specifications of the exogenous variables and parameters over time. In this way can be studied the effect on development of interest rates, price supports, subsidized credit, material and power costs, input supplies and so forth.

5.3. Comparative Statics

For a given year parametric programming can be applied to obtain short run effects on production, investment, etc., of changes in all of the above variables.

5.4. Explaining the Past

While policy makers are concerned most about the future, postmortems are also useful to them. The model's explanation of past development is useful not only as a test of its several components, but also as a net addition to our understanding of what has already happened. Given that we have some confidence in its realism the model can be used to estimate variables about which we have very little data. For example, detailed estimates based on the Punjab model of seasonal labor use and its change over the past two decades are now available. These display a picture of seasonal labor scarcity, a phenomenon that would never have been expected -- at least by academic economists -- a decade ago.

Another example is the detailed chronicle of technological change. The pattern predicted by the model indicates that it is task oriented; that it does not consist of the total replacement of a traditional (bullock, labor intensive) technology by a modernized (tractor, tube-well, labor saving) technology; that it consists rather of a task by task replacement leading to a period of transition during which labor saving and labor using technologies continue to be juxtaposed in a "hybrid technology" whose components depend upon the detailed cost structure of operations and whose proportions change over time. The picture as a whole is not one of balanced growth but rather a counterpoint of development and decay at varying rates and occasionally with switching and reswitching.

NOTES

1. In the field of agricultural analysis in the LDC's two distinct trends of empirical work have emerged. The first has concentrated on econometric analysis of price responsiveness of individual crops (cf. Note 5 below). The second executes the analysis at the sectoral level. The first incorporates such microeconomic variables as input and output prices, crop specific yields and weather indexes while ignoring performance of the sector as a whole. The latter has focused on regional or national indexes of economic activity of the sector as a whole, incorporating output indexes, average wages, labor supply and the interdependence of such sector aggregates with similar indexes of the non-agricultural sector. Cf. JORGENSON [1961], RANIS and FEI [1961] and KELLEY, WILLIAMSON and CHEETAM [1970]. Neither explicitly models the intrasectoral allocation of scarce resources amongst interdependent outputs.

2. In the field of economic development of less developed countries the contributors begin in 1963 with CHENERY [1963], MANNE [1963] and HOLLAND and GELLESPIE [1963].

3. The argument of this section was first presented by us at the seminar of Professors NAKAJIMA and MARUYAMA at Kyoto University in October, 1966.

4. The list is long. The following are representative references: BOEKE [1953], DABASI-SCHWENG [1965], DALTON [1962], FUSFIELD [1957], LEWIS [1955], NAIR [1965], NEAL [1959], OLSON [1960], WHARTON [1963]. Apparently ignorant of or immune to the flood of econometric evidence in the meantime MYRDAL [1968] joined this "traditionalist" school with a vengeance.

5. These studies include those of BAUER and YAMEY [1959], BEHRMAN [1967a], [1967b] and [1968], BROWN [1963], DEAN [1965], FALCON [1964], KAUL [1967], KRISHNA [1963], MANGAHAS [1966], MUYBARTO [1965], and STERN [1962].

6. Very likely the existence of these complications partially explains why the "traditionalists" rejected economic rationality altogether. As we shall see, however, it was not necessary to do so just for the purpose of adding realism to the analysis.

7. The sequence $(\varphi_1,\ldots,\varphi_4)$ is called a lexicographic* or an L^* utility function. Cf. ENCARNACION [1964a], [1964b], ROBINSON and DAY [1971]. Cf. also CHIPMAN [1960], FERGUSON [1965], GEORGESCU-ROEGEN [1954].

8. For an interesting study of the stabilizing effect of cautious behavior cf. MUELLER [1970].

9. This term is due to ROBINSON and DAY [op. cit.]. They also suggest the term Priority* Programming.

10. Ibid.

11. This recalls the work of DUESENBERRY, MODIGLIANI and GEORGESCU-ROEGEN.

12. Hence attitudes toward risk are <u>adaptive</u>, cf. MARSCHAK [1963] but not in a formally Beyesian way, cf. DAY [1971].

13. For example, cf. HEIDHUES [1966], MUDAHAR [1970] and MUELLER [1970] for examples drawn from Germany and India.

14. Cf. DAY and KENNEDY [1970], DAY and ROBINSON [in preparation], and for an elementary discussion DAY and TINNEY [1969].

15. For a profound recognition of this nonanalytical aspect of development, cf. GEORGESCU-ROEGEN [1968] and [1970].

16. However, see LANCASTER [1966] for a beginning in this direction.

17. In the present version of the model the C_i^t's have represented levels of household consumption based on the current population of families home produced foods in a typical subsistence diet. These involve wheat and various pulses. A superior and possible treatment would be based on a nutritional analysis of farm produced foods along the lines developed by SMITH [1971].

18. For a derivation cf. DAY [1969] and SHUKLA [1971].

19. The general reasoning behind such safety constraints is elaborated in DAY [1970b] and DAY [1971]. Alternative versions of this method of accounting for uncertainty include the chance-constrained programming of CHARNES and COOPER [1959], the Safety-First Principle of ROY [1952] and the Focus Loss Principle of SHACKLE [1958]. The last principle has been applied by PETIT and BOUSSARD [1967]. Comparison of these methods with the conventional portfolio approach FREUND [1956] has been made by BOUSSARD [1969]. We use here the form suggested by HENDERSON [1959] cf. below § 2.3 (7).

20. An alternative formulation where short, medium and long term loans are included is being studied.

21. For an analysis of the adoption constraint based on a simple diffusion theory see DAY [1970a].

22. Subsistence (farm produced) consumption depends primarily on family size and cash income. In the original version of the model these amounts were taken to be exogenous. However, in the current version treatment of subsistence follows the text.

23. As noted above (n.7) this form is based on HENDERSON [1959]. It has been used by DAY [1962] and SCHALLER [1963]. An interesting alternative form has been proposed by CIGNO [1971].

24. This idea is discussed in DAY et. al. [1969] and further elaborated in DAY and NELSON [forthcoming]. The flexible accelerator which makes up the right side of expression (38) was used by CHENERY [1952].

25. For the aggregation theory behind this treatment cf. DAY [1963], CIGNO [1971] and QUIRINO and PARIS [1970].

26. The complete methodology, data and estimation procedures used to estimate the model are given in SINGH [1971].

27. Detailed data on field crop technology in the Punjab are available in SINGH, DAY and JOHL [1968]; fertilizer field trial data for estimating yield-fertilizer response functions for various crops were obtained from the Department of Soils, P.A.U., Ludhiana [1965]; harvest prices for outputs and indices of input prices for various inputs were obtained partly from publications of the Economic and Statistical Organization, Government, Punjab [1951-1965] and partly collected in the field; data for estimating household retained (subsistence) consumption functions and cash expenditures and incomes were obtained from publications of the Board of Economic Inquiry, Punjab [1951-1965]; the Indian census data were used to estimate the regional number of farming households, family labor and hired labor, [1965] and Punjab state census data were used to estimate land, machine and animal draft availabilities.

28. Though acreages sown are the aggregate time series we test in our model, in theory <u>any</u> aggregate series for which the observed and predicted values are available can be tested according to these varying criteria.

29. An independent test of each of the null hypotheses (H_0: $\alpha=0$ H_0: $\beta=1$) is chosen over the joint test (H_0: $\alpha=0$, $\beta=1$) because we can expect the standard F tests used for testing such a hypothesis to be extremely biased under conditions of estimate interdependence.

These statistics are helpful in pinpointing systematic errors in the model which once known can be corrected for. If error terms are serially correlated then the assumptions of the simple least squared estimator are violated, and the rejection of our null hypotheses does not mean that the model does not do well, but only that the tests themselves are biased under the conditions.

30. See THEIL [1967, 237-251] and THEIL and MNOOKIN [1965, pp. 34-55] for a detailed description of this statistic. THEIL interprets proportions (shares) that are positive and sum to unity as probabilities [1967]. The statistic has been recently used by PARKS [1969] and by GOLDBERGER and GAMELETSOS [1967] to compare alternative demand models by their ability to predict expenditure shares of various commodities. THEIL and MNOOKIN [1966] also used the statistic first for the same purpose.

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